9^{th} World Mathematics Team Championship 2018

Junior Team Detailed Solutions

1. The digits are: 9, 9, 5 (3 numbers) , 9, 8, 6 (6 numbers), 9, 7, 7 (3 numbers), 8, 8, 7 (3 numbers); all 15 numbers.

Answer: 15

2. If the first answer is "no", then the fake is among the remaining 5. Straightforward search (by dividing each time into to almost equal groups and adding from the first 4 to obtain group of 4) gives three more moves. If the first answer is "yes", the same strategy applies.

Answer: 4

3. Denote the number of children in the original group by n and the sum of their weights by x. Then $\begin{cases} x = 35.2n \\ x + 45.6 = 36(n+1) \end{cases}$ with solution n = 12 and x = 390.

Answer: 12

4. We have $S_{APC} = \frac{1}{2} S_{ABC} = 15 \text{ cm}^2$ and $S_{PMN} = \frac{1}{3} S_{APC} = 5 \text{ cm}^2$.

Answer: 5 cm^2

5. If the fourth number is divisible by 9 then the first number is divisible by 3, so the first condition is irrelevant. Since the second number is divisible by 5 its last digit is either 0 or 5. Therefore the last digit of the fourth number (that is divisible by 9) is either 2 or 7.

The last digit of a positive integer t such that the last digit of $9t$ is either 2 or 7 is 3 or 8. By checking the first values of t = 3, 8, 13 and 18 we see that only t = 18 works. The numbers are: 159, 160,161 and 162 with sum of 642.

Answer: 642

6. The required numbers are all odd numbers divisible by 15 or numbers of the form $30t + 15$. Then $t = 0, 1, \ldots, 66$ and the answer is 67.

Answer: 67

7. There are two choices for each entry, in total $2^4 = 16$.

Answer: 16

8. The four corner cells have two neighbors each so they should be black. The condition of the problem implies that one of the cells (1,2) and (2,1) is white and the other one – black. Without loss of generality let (1,2) be white. Then (2,2) and (1,3) are black. Then (for (1,3) being black) the cell (2,4) is white. By the same way we get the following coloring:

BLACK	WHITE	BLACK	BLACK
BLACK	BLACK	BLACK	WHITE
WHITE	BLACK	BLACK	BLACK
BLACK	BLACK	WHITE	BLACK

Answer: 4

9. Since $\overline{a2018b}$ is divisible by 4 we have b = 0, 4 or 8. Also $a + b + 11$ is divisible by 3 but not by 9. For b = 0 the maximum value of a is 4; for b = 4 the maximum value of a is 9 and for b = 8 the maximum value of a is 5. The solution is achieved for a = 9, b = 4 or a = 5, b = 8 giving $a + b$ = 13.

Answer: 13

10. T = 4. Denote the required time in hours by t. The condition of the problem implies: $\left(1-\frac{t}{3}\right)=4\left(1-\frac{t}{2}\right)$ giving $t=\frac{9}{5}$ hours or 108 minutes.

Answer: 108

11. T = 13. Denote the number of blue balls by n and the sum of the numbers on blue balls by x. Then $\begin{cases} \frac{x}{n}=7 \\ \frac{72-x}{9-n}=10 \end{cases}$ with solution n = 6 and x = 42. The example is:

4, 5, 6, 7, 8, 12 – blue and 9, 10, 11 – red.

Answer: 6

12. T = 4. Since $S_{CDM}=\frac{1}{2}S_{ABCD}=S_{ABD}$ and we have that $S_{ABD}=25$ cm^2. Also $S_{AMD}=\frac{4}{5}S_{ABD}=\frac{4}{5}\times 25=20$ cm^2.

Answer: 20 cm^2

13. T = 642. We have 9 one digit numbers and 90 two digit numbers giving in total 189 digits. When we write next 150 three digit numbers we have 450 more digits. So after these 150 three digit numbers we have 189 + 450 = 639 digits. Therefore, the 642-th digit in the sequence equals the last digit of the 151-st three digit number. The first three digit number is 100, the second one is 101 and so on, the 151-st three digit number is 151 + 99 = 250. The last digit is 0.

Answer: 0

14. S = 16, T = 5. Denote the number of rooms by x. The condition of the problem implies $\frac{5x-2}{2}+2=16$ giving x = 6.

Answer: 6