9th World Mathematics Team Championship 2018

Junior Level Team Round

English Version

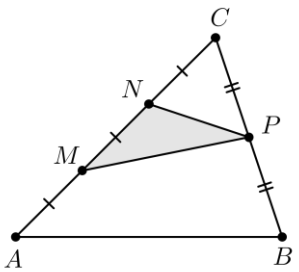
Instruction: This round has 14 questions (**40 minutes**).
Each question is worth 5 points.
No point penalty for submitting wrong answer.

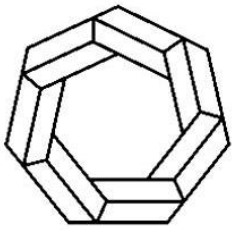
1. Find the number of 3 digit numbers without 0 in their decimal representation with sum of all digits equals to 23.

2. In a pile of 9 coins one is a fake. Suppose we can find out whether the fake coin is among them in any move where we pick four coins. Find the minimum number of moves that guarantee finding the fake coin.

3. The average weight of a group of people is 35.2 kilograms. Albert, who weighs 45.6 kilograms, then joins the group. This raises the average weight of the group to 36 kilograms. How many children were in the original group?

4. Points M, N on the side AC and point P on the side BC of triangle ABC are such that $AM = MN = NC$ and $BP = PC$.
If $S_{ABC} = 30$ cm^2 , find S_{MNP}.





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5. Four consecutive positive integers have the following property:

The first number is divisible by 3

The second number is divisible by 5

The third number is divisible by 7

The fourth number is divisible by 9

Find the least possible sum of these numbers.

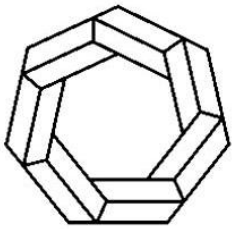
6. How many numbers in the interval [1, 2018] can be expressed as a sum of k consecutive positive integers for each $k = 2$, $k = 3$ and $k = 5$?

7. Find the number of sequences of zeros and ones of length 4.

8. Each cell of a 4×4 table is colored either white or black. Any white cell has exactly 3 black neighboring cells and any black cell has exactly one white neighboring cell. Find the number of white cells. (Two cells are neighbors if they share a common side.)

9. If the number $\overline{a2018b}$ is divisible by 12 but not divisible by 9 find the largest value of $a + b$.

Junior Level Team Round



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***T* = The Solution of Problem #2**

10. Two candles of equal height burn completely for 3 hours and for 2 hour, respectively. The two candles have been lighted simultaneously. After how many minutes the height of one of them is ***T*** times the height of the other?

***T* = The solution of problem #9**

11. The numbers 4, 5, 6, 7, 8, 9, 10, 11 and 12 are written on 9 balls. Every ball is either blue or red. The average of the numbers on blue balls is 7, the average of the numbers on red balls is ***T*** – 3. Find the number of blue balls.

***T* = The Solution of Problem #8**

12. A point *M* on the side *AB* of a parallelogram *ABCD* is such that $AM = \boldsymbol{T} \cdot MB$. If $S_{CDM} = 25$ cm^2 , find S_{AMD}.

***T* = The Solution of Problem #5**

13. All positive integers are written one after another 123456789101112… . Find the digit on the ***T***-th place in this sequence.

***S* = The Solution of Problem #7; *T* = The Solution of Problem #4**

14. A house consists of several rooms. Each room has ***T*** doors. Two of the doors are outside and the remaining doors are between rooms. If the total number of doors is ***S***, how many rooms are in the house?