

9th World Mathematics Team Championship 2018

Junior Level Round 2 Numerical Answers

1	2	3	4	5	6	7	8
1000	1024	2	600	2408	5	4	1

Junior Level Round 2 Detailed Solutions

- All tires traveled 4000 kilometers. The other three travelled 3000 kilometers, 1000 each.
- Since the set $a, 1, 2^1, 2^2, \dots, 2^{10}$ can be split into two sets of equal sums the sum of all elements should be even. Therefore a must be odd. Moreover all numbers from 1 to $2^{11} - 1$ can be expressed as sum of some of $1, 2^1, 2^2, \dots, 2^{10}$ and a is less than 2^{11} . The number of odd numbers less than 2^{11} is $2^{10} = 1024$.
- If $x > -4$ then $(x+4) + (x+5) = (x+1) + (x+2) + (x+3)$ which is possible only if $x+1 > 0$, so all numbers are positive. Analogously, if $x < -1$ all numbers are negative. Therefore all numbers have the same sign. If all are positive then $x + (x+2)$ should be equal to one of the remaining numbers. Therefore $x + (x+2) \leq x+5$ giving $x \leq 3$. On other hand

$$(x+4) + (x+5) = a + b + c \leq (x+1) + (x+2) + (x+3)$$

giving $x \geq 3$. Direct verification shows that 3, 4, 5, 6, 7, 8 works. Analogously, if all are negative the solution is $-8, -7, -6, -5, -4, -3$.

Answer: 2

- There are 65 numbers with first digit 1 and 65 numbers with first digit 8 (for example with first digit 1 the numbers are: $11x, 13x, 14x, 15x, 16x, 17x, 18x - 8$

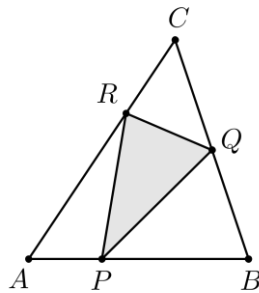
numbers each; $19x - 9$ numbers). With first digit 2, 3, 4, 5, 6 or 7 there are 66 numbers and with first digit 9 there are 74 numbers ($99x - 9$ numbers; $97x, 96x, 95x, 94x, 93x, 92x, 91x - 8$ numbers each and $920 - 9$ numbers). All numbers are $2 \times 65 + 6 \times 66 + 74 = 600$.

5. There are 8 choices for the color of the cell (1,1). If the cell (2,2) is the same color then for each of the other two cells there are 7 choices, altogether $8 \times 7 \times 7$.

If cell (2,2) is of different color from cell (1,1) then there are 7 choices for this color and 6 choices for each of the remaining two cells, altogether $8 \times 7 \times 6 \times 6$.

The answer is $8 \times 7 \times 7 + 8 \times 7 \times 6 \times 6 = 8 \times 7 \times (7 + 36) = 56 \times 43 = 2408$.

6. Since $S_{APR} = \frac{2}{9}S_{ABC} = 4 \text{ cm}^2$, $S_{BPQ} = \frac{1}{3}S_{ABC} = 6 \text{ cm}^2$ and $S_{CRQ} = \frac{1}{6}S_{ABC} = 3 \text{ cm}^2$ we have $S_{APR} = 18 - 4 - 6 - 3 = 5 \text{ cm}^2$.



7. Each one of $\frac{1}{3} + \frac{2}{4} = \frac{5}{6}$ and $\frac{1}{2} + \frac{5}{6} = \frac{4}{3}$ implies 2 solutions (for example the first one implies $a = 1, b = 3, c = 2, d = 4, e = 5, f = 6$ or $a = 2, b = 4, c = 1, d = 3, e = 5, f = 6$).

8. Year 2000 is good for 1981, 2002 is good for 1982, and so on 2016 is good for 1989 and 2018 is good for 2008.

Further, 2001 is good for 1977, 2003 is good for 1978, 2005 is good for 1979, 2009 is good for 1990, 2011 is good for 1991, and so on 2017 is good for 1994. Only 2007 is not good for any other year. This follows from $2007 = \overline{abcd} + a + b + c + d$ noticing that $a = 1$ and $b = 9$ and reducing to $11c + 2d = 97$ with no solution in digits.