9^{th} World Mathematics Team Championship 2018

Junior Level Round 1 Numerical Answers

1	2	3	4	5	6	7	8
C) 1584	**A) 7**	**C) 216**	**A)** $\frac{28}{25}$	**D)** $16\frac{5}{6}$	**E)** $2^3\times3^2\times5^2\times7$	**B)** $\frac{1}{2018}$	**D)** $\frac{5}{8}$

9	10	11	12	13	14	15
B) $\frac{5}{126}$	**D) 8**	**A) 66**	**B) 7**	**C) 9**	**B) 84**	**C)** $\frac{19}{15}$

Junior Level Round 1 Detailed Solutions

1. $1 + 2 - 3 + 4 + 5 - 6 + 7 + 8 - 9 + \ldots + 97 + 98 - 99$

$$= 0 + 3 + 6 + \ldots + 96$$

$$= 3(1 + 2 + \ldots + 32)$$

$$= 3\times\frac{1}{2}\times32\times33=1584$$

2. $1-\cfrac{1}{1+\cfrac{1}{1-\cfrac{1}{1+\cfrac{1}{2}}}}=1-\cfrac{1}{1+\cfrac{1}{1-\cfrac{2}{3}}}=1-\frac{1}{4}=\frac{3}{4}$, giving $p = 3$, $q = 4$, thus $p + q=7$.

3. Take one number 6 from each brackets and group the remaining summands in each brackets to the sum of 12:

(11 + 10 + ... + 1) + (10 + 9 + ... + 2) + (9 + 7 + ... + 3) + (8 + ... + 4) + (7 + 6 + 5) + 6 = $6\times 6+12\times(5+4+3+2+1)=36+180=216$.

4. $\frac{2^3+2^4+2^5}{3^2+4^2+5^2}=\frac{8+16+32}{9+16+25}=\frac{56}{50}=\frac{28}{25}$.

5. A(1,2) + A(1,3) + A(1,4) + A(2,3) + A(2,4) + A(3,4)

$$=\left(\frac{1}{2}+\frac{2}{1}\right)+\left(\frac{1}{3}+\frac{3}{1}\right)+\left(\frac{1}{4}+\frac{4}{1}\right)+\left(\frac{2}{3}+\frac{3}{2}\right)+\left(\frac{2}{4}+\frac{4}{2}\right)+\left(\frac{3}{4}+\frac{4}{3}\right)$$

$$=\left(\frac{2}{1}+\frac{3}{1}+\frac{4}{1}\right)+\left(\frac{1}{2}+\frac{3}{2}+\frac{4}{2}\right)+\left(\frac{1}{3}+\frac{2}{3}+\frac{4}{3}\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right)$$

$$=9+4+\frac{7}{3}+\frac{3}{2}=16\frac{5}{6}$$

6. Since $300=2^2\times 3\times 5^2$ and $2520=2^3\times 3^2\times 5\times 7$ we have

LCM(300,2520) = $2^3\times 3^2\times 5^2\times 7$

7. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)...\left(1-\frac{1}{2018}\right)=\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times...\times\frac{2017}{2018}=\frac{1}{2018}$.

8. The smallest fraction is $\frac{5}{9}$ and the largest fraction is $\frac{9}{8}$ with product of $\frac{5}{8}$.

9. $\frac{2\times 33\times 404\times 5005}{6\times 77\times 808\times 9009}=\frac{3\times 4\times 5}{3\times 7\times 8\times 9}=\frac{5}{126}$.

10. The last digit of
$1\times 2+2\times 3+3\times 4+4\times 5+5\times 6+6\times 7+7\times 8+8\times 9+9\times 10+10\times 11$ equals

the last digit of 2 + 6 + 2 + 0 + 0 + 2 + 6 + 2 + 0 + 0 = 0. By grouping all summands by tens we get that the desired digit equals the last digit of

$$2011\times 2012+2012\times 2013+...+2016\times 2017+2017\times 2018$$

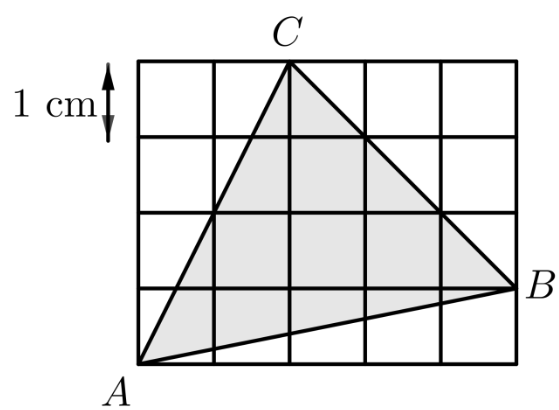
which is the last digit of $1\times 2+2\times 3+3\times 4+4\times 5+5\times 6+6\times 7+7\times 8$ or $2+6+2+0+0+2+6=8$.

11. Since $1452=2^2\times 3\times 11^2$ and $2970=2\times 3^3\times 5\times 11$ we have

GCD(1452,2970) = 66.

12. Since 123:7 = 17.57142857142857… with period of length 6 and $2018=6\times 336+2$ the answer is 7.

13. The area of the rectangle is $4\times 5=20$ cm^2 and the areas of the rectangles having diagonals *AB*, *BC* and *AC* are 2.5 cm^2; 4.5 cm^2 and 4 cm^2, respectively. The area of ABC equals 20 – 2.5 – 4.5 – 4 = 9 cm^2.



14.(2♥0)♥(1♥8) = (0 + 6)♥(8 + 3) = 6♥11 = 66 + 18 = 84

15. It is clear that $a < b$ and $c < d$. Also without loss of generality let $a < c$. Then a is the smallest of the numbers, i.e. $a = 3$. Direct check shows that among the three options ($b = 4$, $c = 5$, $d = 6$; $b = 5$, $c = 4$, $d = 6$ and $b = 6$, $c = 4$, $d = 5$) the least value is achieved for $b = 5$, $c = 4$, $d = 6$, i.e. it is $\frac{3}{5}+\frac{4}{6}=\frac{19}{15}$.