9th World Mathematics Team Championship 2018

Intermediate Level Team Round

English Version

Instruction: This round has 14 questions (**40 minutes**).
Each question is worth 5 points. No point penalty for submitting wrong answer.

1. Given five distinct positive integers such that all possible sums of two of these numbers are distinct. Find the least possible value of the largest of these five numbers.

2. For a positive integer n by $S(n)$ we denote the sum of the digits of n. Find the least value of n for which $S(n)S(n + 1) = 2018$.

3. Given an isosceles triangle ABC with $\angle C = 90°$ and $AC = BC = 6$. Let Y and X be points on AC and BC respectively so that $CY = CX = 2$. If D is the intersection point of AX and BY find the area of $\triangle ADB$.

4. Find the number of words of length 7 composed by two letters a and b such that there are no two adjacent letters a.

5. Some of the cells of 5×5 table are colored. What is the minimum number of cells having even number of colored adjacent cells (two cells are adjacent if they share a common side)?

6. Water is pouring from a spring at a constant speed of 40 liters per minute. Ten elephants drink all the water evenly distributed the amount of water among them. After 1 minute two elephants decided they had drunk enough and left. After one more minute three more elephants left. After the third minute the remaining 5 elephants also left. Find the amount of water each of the last 5 elephants has drunk for the entire 3 minutes.

7. Find the angle between clock arrows in 7 hr 38 min. (Using a 12-hour analog clock.)

8. Point M on the side AB and point N on the side AC of triangle ABC are such that $AN = NM = MC = CB$ and $\angle BAC = 20°$. Find $\angle BNM$.

9. For what value of parameter a the equation $|2x - 1| + |3x - 2| = a - x$ has infinitely many solutions?

T = the answer of Problem #7

10. If $\overline{bcT}$ is divisible by 3, $\overline{Tbc}$ is divisible by 4 and $\overline{cTb}$ is divisible by 5 find c.

T = the answer of Problem #9
11. Find the number of pairs (x, y) of positive integers for which $x + y = Txy - 7$.

T = the answer of Problem #1
12. In a football (soccer) tournament with seven teams every two teams played one game against each other. The total number of points is divisible by $2T$. Find the number of wins in this tournament.

(In football for win the team gets 3 points, for draw gets 1 point and for loss gets 0 points)

T = answer of Problem #3

13. Find the largest positive integer k for which 3^k is a divisor of

$$3 \times 33 \times 333 \times ... \times \underbrace{33...33}_{T}.$$

S = the answer of Problem #6; T = the answer of Problem #8

14. A student runs along a tram route with constant speed. The trams run in the two directions with constant speed through the same intervals. Every S minutes he is overtaken by a tram and every T minutes he meets a tram. Find the ratio of the speed of the student and the speed of the tram.