# $9^{th}$ World Mathematics Team Championship 2018

**Intermediate Relay Detailed Solutions**

**R1-A** Sum of 14 is achieved by 6 + 6 + 2; 6 + 4 + 4; 5 + 5 + 4 and 6 + 5 + 3.

For each of the first three triples the probability $3\times\frac{1}{6^3}$ and for the last one it is $6\times\frac{1}{6^3}$. The probability is $3\times3\times\frac{1}{6^3}+6\times\frac{1}{6^3}=\frac{15}{6^3}=\frac{5}{72}$, giving $p = 5$, $q=72$ with

$$p + q = 77.$$

Answer: 77

$T = 77$

**R1-B** Let $\angle ACB=\gamma=T$, $\angle ABC=\beta$ and $\angle BAC=\alpha$. Since $PM$ and $QM$ are medians in the right triangles $ABP$ and $ABQ$ we have that $BM = PM = QM = AM$. Therefore $\angle BMP=180°-2\beta$ and $\angle AMQ=180°-2\alpha$ implying

$$\angle PMQ=180°-(180°-2\beta)-(180°-2\alpha)=-180°+2(\alpha+\beta)=-180°+2(180°-\gamma)$$
$$=180°-2\gamma=180°-2(77°)=26°.$$

Answer: 26

**R2-A** Since $b + c > a$ we have that $|a-b-c| = b+c-a$ and therefore

$$||a-b-c|-c| + |a-b-c| = |b-a| + b + c - a$$

If $a > b$ the above expression equals $c$ and if $a < b$ it is $2(b - a) + c$. The greatest value equals 2017 and it is achieved for $a = 2012$, $b = 2014$ and $c = 2013$

Answer: 2017

$T = 2017$

**R2-B** The distance travelled by $B$ is two times the distance travelled by $A$. The two points meet for the first time at point $X$ such that the arc $BX$ is two times the arc $AX$. After that they meet at $B$ then at $Y$ then at $X$ and so on.

Since $T = 2017 = 3 \times 672 + 1$ they meet for the last time at $X$.

Answer: 60

**R3-A**

$$\sqrt{3-\sqrt{4+\sqrt{12}}} + \sqrt{3+\sqrt{4-\sqrt{12}}}$$

$$= \sqrt{3-\sqrt{\left(1+\sqrt{3}\right)^2}} + \sqrt{3+\sqrt{\left(1-\sqrt{3}\right)^2}}$$

$$= \sqrt{2-\sqrt{3}} + \sqrt{\sqrt{3}+2} = \sqrt{\frac{\left(\sqrt{3}-1\right)^2}{2}} + \sqrt{\frac{\left(\sqrt{3}+1\right)^2}{2}} = \frac{2\sqrt{3}}{\sqrt{2}} = \sqrt{6}$$

giving $5x^2 = 30$.

Answer: 30

$T = 30$

**R3-B** The induced graph has no triangles. The maximum number of edges in a graph of $T = 30$ vertices without triangles equals $15^2 = 225$. The maximum number of presents is $2 \times 225 = 450$.

Answer: 450