9th World Mathematics Team Championship 2018

Intermediate Level Round 2 Numerical Answers

1	2	3	4	5	6	7	8
750	120	3	130	100	1009	6720	61

Intermediate Level Round 2 Detailed Solutions

1. The kilometers in which one of the tires A and B is a spare tire are 450. The kilometers in which one of the tires C and D is a spare tire are 300. So tire E has been spare tire for 1000 – 450 – 300 = 250 kilometers and it has been in use for 750 kilometers. Such a model is possible for: $ABCE$ – 300 km, $BCDE$ – 450 km and $ABCD$ – 250 km

2. If $\angle ACD = \gamma$ and $\angle BDC = \delta$ then $\gamma + \delta = \angle BAO + \angle ABO = 120^\circ - \angle DAC - \angle DBC = 120^\circ - \gamma - \delta$. Thus $\gamma + \delta = 60^\circ$ and the answer is 120°.

3. After cancelling $a_n^2 = 2a_{n+1} - 1$ the sum is equal to $2a_1 - 1 = 3$.

4. The angle between the altitudes from A and B equals $180^\circ - \gamma$. Therefore $\gamma = 80^\circ$. The angle between the angular bisectors of angle A and angle B equals $90^\circ + \frac{\gamma}{2} = 90^\circ + 40^\circ = 130^\circ$. Depending on the choice of the angle between the altitudes and the angular bisectors all 140°, 150° and 40° are acceptable.

5. Since $2b$ is even we have a carryover from $c + a$. The last digit of $a + c$ is odd and there is no carryover from $2b$. So $a + c = 11, 13, 15$ or 17 and $b = 0, 1, 2, 3$ or 4. Since $a + c = 11, 13, 15$ or 17 for 8, 6, 4 and 2 ways we have 20 options for $a + c$; 5 options for b; all 100.

6. Let $R \neq P$ be a point on the line AB such that $BR = BP$. Triangle PRC is isosceles and PQ is middle segment in ARC. Thus, $PQ = \frac{1}{2}CR = \frac{1}{2}CP = 1009$ cm.

7. Let a_i for $i = 1, 2, 3, 4$ be the number of rings of finger i. Then $a_1 + a_2 + a_3 + a_4 = 5$ for $a_i \geq 0$ has $\binom{8}{3} = 56$ solutions. The five rings can be distributed in $5! = 120$ ways. The answer is $56 \cdot 120 = 6720$.

8. Let the number of problems from day 1 to day 11 inclusive be x; from day 12 to day 14 inclusive be y and from day 15 to day 31 inclusive be z. The condition of the problem gives $x + y = 5 \cdot 14 = 70$ and $y + z = 3 \cdot 20 = 60$. Thus

$$n = x + y + z = 130 - y.$$

Since $0 \leq y \leq 60$ the number of different values of n is 61.