9th World Mathematics Team Championship 2018

Advanced Relay Detailed Solutions

R1-A With no lines we have one region. The first line adds one more region; the second line adds 2 more regions and so on the n-th line adds n more regions. Therefore the total number of regions equals $1+1+2+...+20=1+\frac{20\times 21}{2}=211$.

Answer: 211

T = 211

R1-B If $b-a>2$ we may increase a by 1 and decrease b by 1 having $a+1<b-1$. Then

$$ab+bc+ca=ab+c(\mathrm{T}-c)<(a+1)(b-1)+c(211-c)$$

and the value increases. We conclude that $b-a\le 2$ and the same arguments show that $c-b\le 2$.

If $b-a=2$ and $c-b=2$ then $a+1$, b and $c-1$ increase the value of the expression. Therefore:

1. $b-a=1$ and $c-b=1$ giving $3b=211$ with no solution in positive integers;
2. $b-a=1$ and $c-b=2$ giving $a=69$, $b=70$, $c=72$
3. $b-a=2$ and $c-b=1$ giving $3b=212$ with no solution in positive integers.

Answer: 70

R2-A We can choose the two positions in which a nice number coincides with 12345 in 10 ways. After that we choose the two positions in which a nice number coincides with 23456 in 3 ways. The remaining digit of a nice number can be chosen in 7 ways. All nice numbers are $10 \times 3 \times 7 = 210$.

Answer: 210

T = 210

R2-B Let M be the midpoint of CD. Then CM = 105 cm, OC = 350 cm and from triangle OCM we have OM = $OM = 35\sqrt{91}$ cm. If x is the distance from C to OD we have $x \times 350 = 210 \times 35\sqrt{91}$ giving $x = 21\sqrt{91}$.

Answer: $x = 21\sqrt{91}$ cm

R3-A Since $\frac{2}{a_n} = \frac{1}{a_{n-1}} + \frac{1}{a_{n+1}}$ the reciprocal values form an arithmetic sequence with first term $\frac{1}{3}$ and common difference $\frac{1}{6}$. Therefore $\frac{1}{a_{2018}} = \frac{1}{3} + 2017.\frac{1}{6} = \frac{673}{2}$.

Answer: 673

T = 673

R3-B We can delete three letters *abc* or *aaa* or *bbb* or *ccc*. We are left with *a*, *aa*, *ab*, *aab* or *aabb*. Only in *aab* it is not possible to achieve divisibility by 3. So, good are only words of length not divisible by 3.

There are 448 numbers divisible by 3 (from 3×225 to 3×672) in the interval [673,2018]. All numbers are 2018 – 672 = 1346. Not divisible by 3 are 1346 – 448 = 898.

Answer: 898