9^{th} World Mathematics Team Championship 2018

Advanced Level Round 1 Numerical Answers

1	2	3	4	5	6	7	8
C) 2	**A) –5**	**D)** $\frac{\sqrt{15}}{8}$	**D) 2520**	**E) 15 cm**	**C) 576**	**D) 8**	**B) 2**

9	10	11	12	13	14	15
E) 5	**B)**	**B) 5**	**A) 42**	**B) [3,4]**	**A)** $\frac{4a}{3(2-a)}$	**D)** $-\frac{3}{2}$

Advanced Level Round 1 Detailed Solutions

1. $\sqrt{9}+\sqrt{\left(3-\sqrt{11}\right)^2}+\sqrt[3]{\left(2-\sqrt{11}\right)^3}=3+\sqrt{11}-3+2-\sqrt{11}=2$.

2. Note that $2^{x-1}\times 5^{x-1}=10^{x-1}$ and $0.1\times 10^{2x+5}=10^{2x+4}$. It follows that $x-1=2x+4$, or $x=-5$.

3. From $\sin\alpha=\pm\sqrt{1-\cos^2\alpha}=\pm\frac{\sqrt{15}}{4}$ and $\sin\alpha<0$ we obtain $\sin\alpha=-\frac{\sqrt{15}}{4}$.

Thus, $\sin 2\alpha=2\sin\alpha\times\cos\alpha=\frac{\sqrt{15}}{8}$.

4. There are 10 choices for the leader, 9 for the deputy leader and $\binom{8}{2} = 28$ for the two members. For the whole group there are $10 \times 9 \times 28 = 2520$ choices.

5. Denote the length of the unknown side by *x*. The altitude to the side of length *x* cm is less than 9 cm and therefore it is the altitude of length 6 cm. Then $S = 9 \times 10 = 6 \times x$ giving *x* = 15 cm.

6. All arrangements of the six girls are 6! = 720. The arrangements for which the three Chinese girls are sitting together next to each other are $4! \times 3! = 144$. Therefore the answer is 720 – 144 = 576.

7. Since $\sqrt{3-\sqrt{5}} = \sqrt{\frac{6-2\sqrt{5}}{2}} = \frac{\sqrt{5}-1}{\sqrt{2}}$ we have

$$\left(3+\sqrt{5}\right)\left(\sqrt{10}-\sqrt{2}\right)\sqrt{3-\sqrt{5}} = \left(3+\sqrt{5}\right)\left(\sqrt{5}-1\right)^2 = 2\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right) = 8.$$

8. For *u* = *x* + *y* and *v* = *xy* the system becomes $\left| \begin{array}{l} u+v=7 \\ u \times v = 12 \end{array} \right.$ with solutions *u* = 3, *v* = 4 and *u* = 4 and *v* = 3. The first pair doesn't lead to solutions for *x* and *y* and the second one gives two solutions. /

9. Since the number of 2-digit numbers having only odd digits is $5 \times 5 = 25$ we have that each odd digit appears as unit digit 25 times. Thus the unit digit of the sum of all 3 digit numbers having only odd digits equals the unit digit of $\left(1+3+5+7+9\right) \times 25 = 25 \times 25$, i.e. it is 5.

10. Since the range is $\left[-\frac{1}{8},\infty\right)$ we have that $a > 0$. Then the minimum value of $y = ax^2 + c$ equals c , thus the answer is B).

11. Using that $MA^2 + MC^2 = MB^2 + MD^2$ we obtain $29 = 4 + MD^2$ giving $MD = 5$

12. Since $1+i\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ we have

$$\left(1+i\sqrt{3}\right)^{21} = \left(2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right)^{21} = 2^{21}\left(\cos 7\pi + i\sin 7\pi\right) = -2^{21}.$$

Thus $a^2 = 2^{42}$ and $\log_2 a^2 = \log_2 2^{42} = 42$.

13. Since $\left(x^2 - 5x + 4\right) = \left(x-1\right)\left(x-4\right)$ and $x \geq 3$ the solution is [3,4].

14. $\log_2 \sqrt[3]{625} = \frac{1}{3}\log_2 625 = \frac{4}{3}\log_2 5$

$$= \frac{4}{3} \times \frac{\log_{10} 5}{\log_{10} 2} = \frac{4}{3} \times \frac{a}{2\log_{10} 2}$$

$$= \frac{4}{3} \times \frac{a}{\log_{10} 4} = \frac{4}{3} \times \frac{a}{\log_{10} \frac{100}{25}} = \frac{4}{3} \times \frac{a}{2-a}.$$

15. By inspection we find that the solution is $-\frac{3}{2}$. It is also possible to use that

$$\frac{x}{x^2+2}\sqrt{1+\frac{x^4+4}{4x^2}} = \frac{x}{x^2+2}\sqrt{\frac{\left(x^2+2\right)^2}{(2x)^2}} = \frac{x}{x^2+2}\,\frac{x^2+2}{2|x|} = \frac{x}{2|x|} = \begin{cases} \frac{1}{2} & \text{for } x > 0 \\ -\frac{1}{2} & \text{for } x < 0 \end{cases}.$$