9th World Mathematics Team Championship 2018

Advanced Level Individual Round 1

English Version

Instruction: This round has 15 questions (**20 minutes**).
Each question is worth 2 points.
No point penalty for submitting wrong answer.
Blank answer will be assigned 0.5 point.

1. Find the value of $\sqrt{9}+\sqrt{\left(3-\sqrt{11}\right)^2}+\sqrt[3]{\left(2-\sqrt{11}\right)^3}$.

A) $8-2\sqrt{11}$ B) 4 C) 2 D) $2\sqrt{11}-2$ E) $2\sqrt{11}$

2. Find x if $2^{x-1}\times 5^{x-1}=0.1\times 10^{2x+5}$.

A) –5 B) –4 C) –3 D) 3 E) 5

3. If $\cos\alpha=-\frac{1}{4}$ and $\pi\le\alpha\le\frac{3\pi}{2}$, find $\sin 2\alpha$.

A) $\frac{\sqrt{15}}{4}$ B) $-\frac{\sqrt{15}}{4}$ C) $-\frac{\sqrt{15}}{8}$ D) $\frac{\sqrt{15}}{8}$ E) $\frac{\sqrt{15}}{16}$

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4. In how many ways one can choose a group of leader, deputy leader and two members from a group of 10 persons?

A) 2500 B) 2400 C) 5040 D) 2520 E) 210

5. One of the sides of a parallelogram equals 9 cm and the two altitudes are 6 cm and 10 cm. Find the length of the other side.

A) 5.4 cm B) 13 cm C) 8 cm D) 7 cm E) 15cm

6. In how many ways 3 Bulgarian girls and 3 Chinese girls can sit in a bench such that the three Chinese girls are not sitting together next to each other.

A) 144 B) 720 C) 576 D) 625 E) 216

7. Find $\left(3+\sqrt{5}\right)\left(\sqrt{10}-\sqrt{2}\right)\sqrt{3-\sqrt{5}}$.

A) $\sqrt{3}-\sqrt{5}$ B) $3\sqrt{2}$ C) 5 D) 8 E) 10

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8. The number of solutions of $\begin{vmatrix} x+xy+y=7 \\ x^2y+xy^2=12 \end{vmatrix}$ is:

A) 1 B) 2 C) 4 D) 0 E) 3

9. The unit digit of the sum of all 3 digit numbers having only odd digits equals:

A) 1 B) 2 C) 3 D) 4 E) 5

10. For what values of a and c the range of the function $y=ax^2+c$ is the interval $\left[-\frac{1}{8},\infty\right)$?

A) $\begin{vmatrix} a=-\frac{1}{8} \\ c>0 \end{vmatrix}$ B) $\begin{vmatrix} a>0 \\ c=-\frac{1}{8} \end{vmatrix}$ C) $\begin{vmatrix} a<0 \\ c=-\frac{1}{8} \end{vmatrix}$ D) $\begin{vmatrix} a=-\frac{1}{8} \\ c=0 \end{vmatrix}$ E) $\begin{vmatrix} a=0 \\ c=-\frac{1}{8} \end{vmatrix}$

11. A rectangle $ABCD$ and a point M are such that $MA=2\sqrt{5}$, $MB=2$ and $MC=3$ find MD.

A) $\sqrt{5}$ B) 5 C) $2\sqrt{5}+1$ D) $2\sqrt{5}-1$ E) $2\sqrt{6}$

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12. If $\left(1+i\sqrt{3}\right)^{21} = a+bi$ for a and b real numbers find $\log_2 a^2$.

A) 42 B) 21 C) 48 D) 63 E) –48

13. The solution of $\left(x^2-5x+4\right)\sqrt{x-3} \le 0$ is:

A) [3,5] B) [3,4] C) [1,4] D) $[3,\infty)$ E) (3,4)

14. If $a = \log_{10} 25$ find $\log_2 \sqrt[3]{625}$.

A) $\frac{4a}{3(2-a)}$ B) $\frac{4a}{3(a+2)}$ C) $\frac{3a}{2(3-2a)}$ D) $\frac{6a}{3a+4}$ E) $\frac{6a}{3a-4}$

15. Solve $\frac{x}{x^2+2}\sqrt{1+\frac{x^4+4}{4x^2}} = x+1.$

A) $-\frac{1}{2}$ B) $\frac{3}{2}$ C) $\frac{1}{2}$ D) $-\frac{3}{2}$ E) –2